

Roll No.

Total No. of Pages : 02

Total No. of Questions : 09

B.Sc. (Non Medical) (Sem-1)
DIFFERENTIAL CALCULUS

Subject Code : BSNM-105-18

M.Code : 75746

Date of Examination : 13-06-2023

Time : 3 Hrs.

Max. Marks : 50

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying ONE marks each.
2. SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.
3. SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.

SECTION-A

1. Answer briefly :

- a) State Cauchy's general Principle of convergence.
- b) Define bounded sequence.
- c) Define continuity of a function in an interval.
- d) Find $\lim_{x \rightarrow \infty} (7x^3 + 8x^2 + 5x - 7)$
- e) Find the derivative of $\sin 2x$.
- f) State homogeneous function.
- g) State composite function.
- h) Define uniform continuity
- i) State Taylor's theorem.
- j) If $f = x^2 + xy$, Find $\frac{\partial f}{\partial x}$.

SECTION-B

2. Prove that the sequence $\left\{ \left(1 + \frac{1}{n}\right)^n \right\}$ is bounded.

3. Discuss the function defined by

$$f(x) = \begin{cases} 1/q & \text{when } x \text{ is rational } \frac{p}{q} \neq 0 \text{ in lowest terms} \\ 0 & \text{when } x \text{ is irrational and } x = 0 \end{cases}$$

As regards its continuity

4. Find $\frac{dy}{dx'}$ when $y = a(\cos t + \sin t)$, $y = a(\sin t - t \cos t)$

5. If $x = r \cos \theta$, $y = r \sin \theta$, find $\frac{\partial(x, y)}{\partial(r, \theta)}$ and $\frac{\partial(r, \theta)}{\partial(x, y)}$.

6. If $u = x \log xy$ and $x^3 + y^3 + 3xy = 0$, find $\frac{du}{dx}$.

SECTION-C

7. Show that $\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}$, $0 < u < v$ and deduce

$$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}.$$

8. If $U = \sin^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$, Prove that $x^2 \frac{\partial^2 U}{\partial x^2} + 2xy \frac{\partial^2 U}{\partial x \partial y} + y^2 \frac{\partial^2 U}{\partial y^2} = \frac{-(\sin u \cos 2u)}{4 \cos^3 u}$.

9. a) Discuss the continuity of the function $f(x) = \lim_{n \rightarrow \infty} \sum_r^n = 1 \frac{[2rx]}{n^2}$, where $[]$ denotes the greatest integral function.

b) Evaluate $\lim_{x \rightarrow a} \left(2 - \frac{a}{x}\right)^{\tan \frac{\pi x}{2a}}$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.