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Total No. of Questions : 07

B.Sc. (CS) (Sem.–6)

LINEAR ALGEBRA

Subject Code : BCS-602

M.Code : 72782

Date of Examination : 22-05-2023

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SIX questions carrying TEN marks each and students have to attempt any FOUR questions.**

SECTION-A

1 Answer the followings in short :

a. Check the system of vector $[-1, 2, 2], [2, -3, 1], [10, -1, 0]$, are linearly dependent or not.

b. What is meant by vector space? Give two examples.

c. What is meant by Basis of a vector space?

d. Examine whether or not (1,1,1), (1, 2, 3), (2, -1,1) is basis of R^3.

e. Construct two subspaces of $R^4(R)$ such that; dim $W_1 = 2$, dim $W_2 = 2$, dim $(W_1 \cap W_2) = 1$.

f. What is meant by Linear Transformation?

g. Show that there is no non-singular linear transformation from R^4 to R^3.

h. Explain Isomorphic spaces with two examples.

i. Find the matrix representation of $T: R^2 \rightarrow R^2$ defined as $T(x, y) = (3x - 4y, x + 5y)$ with respect to the basis $B = \{(1, 0), (0, 1)\}$.

j. Explain Invariant subspace with two examples.

SECTION-B

2. Show that V = $\{i\beta \mid \beta \in R\ (Reals)\}$ is a vector space over field R where the addition of the space, and the scalar multiplication of the elements of V by those of R are respectively, the addition of complex numbers and the multiplication of a real number with a complex number.

3. Prove that the union of two subspaces is a subspace if one of them is a subset of the other.

4. Prove that any two basis of a finite dimensional vector space have same number of elements.

5. V be vector space of 2 × 2 matrices over R and P = $\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$, define $T: V \to V$ be a linear transformation as $(A) = PA\ \forall\ A \in V$; Find a basis and dimension of null space of T and Range space of T.

6. Find the matrix representation of linear operator $T: R^3 \to R^3$ defined by

   $T(x, y, z) = (3x + z, —2x + y, —x + 2y + 4z)$ relative to the basis B = (1,0,1), (-1,2,1), (2,1,1).

7. $B_1 = \{v_{1,} v_{2,} \dots \dots, v_n.\}$ and $B_2 = \{w_{1,} w_{2,} \dots \dots, w_n\}$ be two ordered basis of vector space V(F) and P is the transition matrix from $B_1 = \{v_i\}$ to $B_2 = \{w_i\}$.

   Prove that P[v; B_2] = P[v; B_1] and [v; B_2] =P^{-1} [v; B_1] for all $v \in V$.

NOTE : Disclosure of identity by writing mobile number or making passing request on any page of Answer sheet will lead to UMC case against the Student.