Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages : 02**

Total No. of Questions : 09

B.Sc. (Non Medical) (Sem.–6)

MODERN ALGEBRA

Subject Code : BSNM-605-18

M.Code : 79497

Date of Examination : 11-07-22

Time : 3 Hrs. **Max. Marks : 50**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying ONE marks each.**
2. **SECTION-B contains FIVE questions carrying FIVE marks each and students have to attempt any FOUR questions.**
3. **SECTION-C contains THREE questions carrying TEN marks each and students have to attempt any TWO questions.**

SECTION-A

1. Write briefly :

a) Define a quotient group.

b) Give an example of non-abelian group.

c) Is it possible that an abelian group has a non-abelian subgroup? Give example.

d) Let G be a group and e be the identity element of G. Define a mapping $f : G \to G$ such that $f(a) = e$ for all $a \in G$. Prove that f is endomorphism.

e) Let S = $\{a\ b\ c\}$. Write down even permutations.

f) Give an example that a subring of a ring with unity may fail to be a ring with unity.

g) Define integral domain.

h) How many generators are there of cyclic group of order 10?

i) Give an example of prime ideal which is not a maximal ideal.

j) Prove that every homomorphic image of a commutative ring is commutative.

SECTION-B

2. State and prove Lagrange's theorem.

3. If R is a ring with identity such that $(xy)^2 = x^2y^2$ for all $x \in$ R, $y \in$ R, show that R is commutative.

4. Show that the mapping f : C $\rightarrow$ R such that $f(x + iy) = x$ is a homomorphism of the additive group of complex numbers onto the additive group of real numbers.

5. Let R is an integral domain with unity. If R′ is an integral domain such that f : R $\rightarrow$ R′ is an homomorphism and ker $f \neq$ R; Show that f (l) is unity of R′.

6. Let $\left\{\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & g \end{bmatrix} : a,b,c,d,e,f,g \in Z\right\}$ be a ring. Show that A = $\left\{\begin{bmatrix} 0 & 0 & a \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} : a \in Z\right\}$ is not an ideal of R.

SECTION-C

7. Let A(s) denotes the set of all permutations on a non-empty set S. Then A(S) forms a group under the operation of composition of maps. Moreover if S contains n elements then the group A(S) contains $n!$ elements.

8. An ideal of a ring of integers is maximal iff it is generated by some prime integers.

9. If H and K are two normal subgroups of a group G such that H $\subseteq$ K. Then G/K $\cong \dfrac{G/H}{K/H}$.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.