Roll No. | | | | | | | | | | | | Total No. of Pages : 02

Total No. of Questions : 07

B.Sc. (Computer Science) (Sem.–6)

REAL ANALYSIS

Subject Code : BCS-601

M.Code : 72781

Date of Examination : 04-07-22

Time : 3 Hrs. **Max. Marks : 60**

INSTRUCTIONS TO CANDIDATES :

1. **SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.**
2. **SECTION-B contains SIX questions carrying TEN marks each and students have to attempt any FOUR questions.**

SECTION–A

1. Write briefly :

a) State Abel's theorem

b) Determine the radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n!}$.

c) Show that $\sum_{n=1}^{\infty} n^2 x^n$ is uniformly convergent in [–α, α], when 0 < α < 1.

d) Show that the series for which $S_n(x) = \frac{1}{1+nx}$ can be integrated term by term on [0, 1], though it is not uniformly convergent on [0, 1].

e) State Cauchy's General Principle of uniform convergence.

f) Show that an analytic function with constant real part is constant.

g) For the conformal transformation $w = z^2$, find the coefficient of magnification at $z = 1 + i$.

h) State analytic function.

i) Prove that $u = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic function.

j) State Dirichlet's conditions.

SECTION-B

2. Prove that the series obtained by integrating and differentiating power series term by term has the same radius of convergence as the original series.

3. Find the exact interval of absolute convergence and of uniform convergence of the following series

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots\ldots$$

4. Show that $\sum \frac{1}{n^3 + n^4 x^2}$ is uniformly convergent for all real x and that it may be differentiated term by term.

5. Show that under the transformation w = $\frac{z-i}{z+i'}$ real axis in the z plane is mapped into the circle $|\, w \,| = 1$. Which portion of the z-plane corresponds to the interior of the circle?

6. Show that in the interval (0, 1) $\cos \pi x = \frac{8}{\pi}\sum_{n=1}^{\infty} \frac{n}{4n^2 - 1} \sin 2n\pi x$

7. Prove that the function $|z|^2$ is continuous everywhere but nowhere differentiable except at origin. Discuss the analyticity of the function $(z) = z\bar{z}$.

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.