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Total No. of Pages : 03

Total No. of Questions : 18

**M.Sc. (IT) / MCA (Sem.-3)**  
**DISCRETE STRUCTURES & OPTIMIZATION**  
Subject Code : PGCA-1917  
Paper ID : 78393

Time : 3 Hrs.

Max. Marks : 70

**INSTRUCTIONS TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying TEN marks each.
4. Select atleast TWO questions from SECTION - B & C.

**SECTION-A**

Answer briefly :

1. If  $A = \{1, 2, 3\}$ ,  $B = \{3, 4, 5\}$ ,  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ . Find  $A \cup B$ ,  $A \cap B$ ,  $A - B$ ,  $\overline{A}$ .
2. Solve  $a_n - 5a_{n-1} + 6a_{n-2} = 0$ , where  $a_0 = 2$ ,  $a_1 = 5$ .
3. Find the number of functions from an  $m$  element set to an  $n$  element set.
4. Find  $x$  and  $y$  if  $(y - 2, 2x + 1) = (x - 1, y + 2)$ .
5. Let  $f$  be a function with domain  $A$  and co-domain  $B$ . Let the relation  $k \subseteq A \times A$  is defined on  $A$  as  $(x, y) \in k$  iff  $f(x) = f(y)$ . Show that  $k$  is an equivalence relation.
6. A graph  $G$  has 21 edges, 3 vertices of degree 4 and other vertices are of degree 3. Find the number of vertices in  $G$ .
7. Define the term simple graph and connected graph. Give one example of each.
8. Prove that any two right (left) cosets in a group  $G$  are either disjoint or equal.
9. Let  $R$  and  $R'$  be two rings. Define  $\theta : R \rightarrow R'$  by  $\theta(a) = 0 \forall a \in R$ . Show that  $\theta$  is a ring homomorphism.
10. If  $(B + \dots)$  is a Boolean algebra, then show that  $a + a.b = a \forall a, b \in B$ .

## SECTION-B

11. i) Let A, B and C are any sets, prove that  $A \cup B = A \cup C$  and  $A \cap B = A \cap C \Rightarrow B = C$ .
- ii) In a survey of 60 people, it was found that 25 read Newsweek magazine, 26 read Times, 26 read Fortune, 9 read both Newsweek and Fortune, 11 read both Times and Newsweek, 8 read both Times and Fortune, 3 read all three magazines. Then find
- a) The number of people who read at least one of the three magazines?
- b) The number of people who read exactly one magazines?
12. i) Find a particular solution of  $S(n) - 4S(n-1) + 4S(n-2) = 2^n$ .
- ii) Find the generating function of  $S(n+2) = S(n+1) + S(n) = 0$  where  $S(0) = S(1) = 1$ , for  $n \geq 0$ .
13. i) How many different reflexive, symmetric relations are there on a set with three elements?
- ii) Show that maximum number of edges in a graph with  $n$  vertices and no multiple edges are  $\frac{n(n-1)}{2}$ .
14. i) Define Eulerian path. Give an example.
- ii) Let  $\theta : R \rightarrow R'$  be a ring homomorphism from a ring  $R$  to the ring  $R'$ . Show that  $\text{Ker } \theta$  is a subgroup of  $R$  under addition.

## SECTION-C

15. i) For sets A and B, prove that  $A \cup B = (A \setminus B) \cup (B \setminus A) \cup (A \cap B)$ .
- ii) Give an example of a relation which is anti symmetric and transitive, but neither reflexive nor symmetric.
16. i) The solution of recurrence relation  $c_0 a_r + c_1 a_{r-1} + c_2 a_{r-2} = f(r)$  is  $3^r + 4^r + 2$ . Given  $f(r) = 6$  for all  $r$ , determine  $c_0, c_1, c_2$ .
- ii) Define the Fibonacci sequence and find its closed form expression.

17. i) Prove that every subgroup of an abelian group is normal.
- ii) Prove that a Boolean algebra cannot have three elements.
18. i) Prove that there does not exist a graph with 5 vertices with degrees equal to 1, 3, 4, 2, 3 respectively.
- ii) If  $G$  is a connected graph and every vertex of  $G$  has even degree then prove that  $G$  has an Euler Circuit.

**NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.**