

Roll No.

Total No. of Pages : 03

Total No. of Questions : 18

B.Tech. (CSE/IT) (2018 & Onwards)/  
(Artificial Intelligence & Machine Learning / Cyber Security) (Sem.–1)

**MATHEMATICS-I**

Subject Code : BTAM-104-18

M.Code : 75362

Time : 3 Hrs.

Max. Marks : 60

**INSTRUCTIONS TO CANDIDATES :**

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions EACH from SECTION - B & C.

**SECTION-A**

1. Discuss the applicability of Rolle's theorem for  $f(x) = x(2+x)e^{-\frac{x}{2}}$  in  $(-3, 0)$ .
2. Define Gamma function ( $\Gamma(x)$ ) and show that  $\Gamma(x) = (x-1)!$ , when  $x$  is an integer greater than 1
3. Expand  $\sin x$  in powers of  $x - \frac{\pi}{2}$
4. Compute  $AB - BA$  if  $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$
5. Find rank of the matrix  $\begin{bmatrix} 2 & 4 & 2 \\ 1 & 2 & 1 \\ 5 & 10 & 5 \end{bmatrix}$ .
6. Define kernel of a linear transformation.
7. Prove that a set of vectors zero vector is always linearly dependent.
8. Define orthogonal matrix.

9. If  $\alpha$  is an eigen value of a non singular matrix A the show that  $\alpha^{-1}$  is eigen value of  $A^{-1}$ .

10. Find the product of eigen values of  $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

### SECTION-B

11. a) Evaluate the integral  $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$  if it exists.

b) Evaluate the limit  $\lim_{x \rightarrow 0} \left( \frac{x2^x}{2^x - 1} \right)$

12. Prove that  $\beta(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$ , where  $\beta(x, y)$  and  $\Gamma(x)$  denote beta and gamma functions respectively.

13. a) Find the inverse of the matrix  $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$  using Gauss Jordan method.

b) Find the rank of the matrix  $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

14. Solve the following system of equations Cramer's rule

$$x + y + z = 6, \quad x - y + 2z = 5, \quad 3x + y + z = 8$$

### SECTION-C

15. Let  $\mathbb{R}^+$  be the set of all +ve real numbers. Operations vector addition '+' and scalar multiplications '!' are defined as

$u + v = uv, u, v \in \mathbb{R}^+, \alpha.u = u^\alpha, \alpha \in \mathbb{R}, u, \in \mathbb{R}^+$ . Determine whether  $\mathbb{R}^+$  form a real vector space or not?

16. a) Examine whether the set of vectors  $\{(3, 2, 1), (1, 1, 1), (-1, 0, 1)\}$  in  $V_3$  forms a basis or not?
- b) Check whether the transformation  $T : V_3 \rightarrow V_2$  defined by  $T(x, y, z) = (x - y, x + y)$  represent a Linear transformation or not?
17. Find the Eigen values and Eigen vectors for the matrix.

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

18. a) Prove that any square matrix can be written as sum of symmetric and skew-symmetric matrices.
- b) Prove that similar matrices have same characteristic polynomial.

**NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.**