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Total No. of Pages : 03

Total No. of Questions : 18

B.Tech.(CE/ME/ECE/EE) (2018 & Onward) (Sem.–1)

MATHEMATICS-I

Subject Code : BTAM-101-18

M.Code : 75353

Time : 3 Hrs.

Max. Marks : 60

INSTRUCTIONS TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. Discuss the applicability of Rolle's theorem on $(x) = x + \frac{1}{x}$ on $\left[\frac{1}{2}, 2\right]$
2. Find Taylor's series of $f(x) = \sin x$ about $x = \frac{\pi}{4}$
3. Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x+y}{x-y}$ does not exist.
4. Find local maxima, local minima, and saddle point for $f(x, y) = x^2 + xy + 3x + 2y + 5$
5. Evaluate $\int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dz dy dx$
6. State Leibniz's test for convergence of an alternating series.
7. Find the value of b for which $1 + e^b + e^{2b} + e^{3b} + \dots = 9$.
8. Find rank of the matrix $\begin{bmatrix} 1 & 4 & 2 \\ -1 & -2 & 1 \\ 9 & 34 & 15 \end{bmatrix}$
9. Define orthogonal matrix.
10. State rank-nullity theorem.

SECTION-B

11. a) Using Mean value theorem show that $|\sin b - \sin a| \leq |b - a|$ for any numbers a and b .
- b) Use L'Hospital's rule to find the limit $\lim_{x \rightarrow 0} \left(\frac{\sin x - x}{x^3} \right)$.
12. a) Evaluate the integral $\int_{-\infty}^2 \frac{2dx}{x^2 + 4}$, if it exists.
- b) Find the area of the surface generated by revolving the curve $y = x^3$, $0 \leq x \leq \frac{1}{2}$ about the x -axis.
13. a) If $u = f(y - z, z - x, x - y)$, then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.
- b) Find the tangent plane and normal line of the surface : $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $(1, 2, 4)$
14. a) Find the area of the region bounded between the curve $x = -y^2$ and the lines $y = x + 2$.
- b) Evaluate the integral $\int_0^\pi \int_x^\pi \frac{\sin y}{y} dx dy$, by changing the order of integration.

SECTION-C

15. a) Discuss the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{6n}{(2n-1)(2n+1)}$
- b) Discuss the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{8 \tan^{-1} n}{n^2 + 1}$.
16. For the series $\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$ find
- a) For what values of x does the series converge absolutely?
- b) For what values of x does the series converge conditionally?
- c) Find the interval of convergence.

17. For what values of λ the given system of equations possess (a) unique solution
b) infinite solution consistent or not? $3x - y + 4z = 3$, $x + 2y - 3z = -2$, $6x + 5y + \lambda z = -3$.
Find the solution in each case.
18. Find the eigen values and eigen vectors of the following matrix :

$$A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$$

NOTE : Disclosure of Identity by writing Mobile No. or Making of passing request on any page of Answer Sheet will lead to UMC against the Student.