

Roll No.

Total No. of Pages: 03

Total No. of Questions: 09

MCA (2015 & Onwards) (Sem.– 2)
MATHEMATICAL FOUNDATIONS OF COMPUTER SCIENCE
M Code: 72876
Subject Code: MCA-201
Paper ID: [72876]

Time: 3 Hrs.

Max. Marks: 60

INSTRUCTIONS TO CANDIDATES:

1. **SECTIONS-A, B, C & D** contains **TWO** questions each carrying **TEN** marks and students have to attempt any **ONE** question from each **SECTION**.
2. **SECTION-E** is **COMPULSORY** consisting of **TEN** questions carrying **TWENTY** marks in all.
3. **Use of non-programmable calculator is allowed.**

SECTION A

1. Prove by mathematical induction that for any natural number n : 10

$$1^3 + 2^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

2. Give comparison between Eulerian path and Hamiltonian path. Justify with example. 10

SECTION B

3. Assume $p(x)$, $q(x)$ and $r(x)$ denote the following statements: 10

$$p(x) : x^2 - 8x + 15 = 0$$

$$q(x) : x \text{ is odd}$$

$$r(x) : x > 0$$

For the domain of all integers, determine the truth-value of each of the following statements. If a universal statement is false, give a counterexample. If an existential statement is true, give an example.

a) $\forall x[p(x) \rightarrow q(x)]$

b) $\exists x[q(x) \rightarrow p(x)]$

c) $\exists x[r(x) \rightarrow p(x)]$

- d) $\forall x[\neg q(x) \rightarrow \neg p(x)]$
- e) $\forall x[(p(x) \vee q(x) \rightarrow r(x)]$
4. a) Let $R = \{(1,2), (2,3), (3,1)\}$ and $A = \{1, 2, 3\}$, find the reflexive, symmetric and transitive closure of R, using
- Composition of relation R.
 - Composition of matrix relation R. 5
- b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$. Also, compute its rank. 5

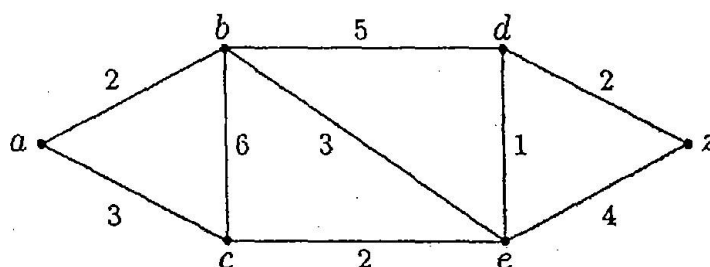
SECTION C

5. Explain the terms 10
- connected graph
 - tree
 - spanning tree
 - Bipartite Graph
 - Countable Sets
6. Solve the following system by using the Gauss-Jordan elimination method. 10
- $$x + y + z = 5$$
- $$2x + 3y + 5z = 8$$
- $$4x + 5z = 2$$

SECTION D

7. a) Using the laws of set theory, simplify each of the following :
- $A \cap (B - A)$ 4
 - $\overline{(A \cup B) \cap C} \cup \overline{B}$
- b) Prove that a planar graph G is 5-colorable. 6

8. Use Prim's algorithm to find a minimum spanning tree in the following weighted graph. Use alphabetical order to break ties. Write the algorithm also. 10



SECTION E

9. Write briefly:
- Consider $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find $C \times B \times A$.
 - Determine whether function $f(x) = -3x^2 + 7$ is a bisection from \mathbb{R} to \mathbb{R} .
 - Compute the rank of matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$.
 - Draw truth table of $\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$.
 - Give an example of graph that has neither an Eulerian circuit nor a Hamiltonian circuit.
 - When a well-formed formula is called as tautology? Give example.
 - Prove that $A - B = A \cap \bar{B}$.
 - If A and B are two equivalent matrices, then show that $\text{rank } A = \text{rank } B$.
 - What is the significance of graph coloring?
 - Give the justification whether the sentence "What is the weather today?" is a proposition or not.